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Drying induced stresses estimated on the base of elastic and viscoelastic models

Jacek Banaszak*, Stefan J. Kowalski

Poznan University of Technology, Institute of Technology and Chemical Engineering, pl. Marii Sklodowskiej-Curie 2, 60-965 Poznan, Poland

Abstract

The main subject of this paper is to study how the chosen constitutive model of a saturated porous body undergoing drying process influences the numerically estimated drying induced stresses and deformations. To compare the results one assumes the material of a convectively dried cylinder to be both elastic and viscoelastic. One states a significant difference between the results obtained for both these models, particularly for the drying induced stresses. © 2002 Published by Elsevier Science B.V.

Keywords: Maxwell model; Constant drying rate period; Drying induced stresses; Clay

1. Introduction

Nowadays, a good quality of dried products is the one of the most important tasks for the drying process engineers. The cracks and splits occurring on the external surfaces of the products are objectionable. To preserve the dried material against the structure destroy, the knowledge on mechanical phenomena occurring inside a body during drying processes is necessary. The considerations in this work are referred to the drying induced stresses that directly influence the fracture of the dried material.

In order to make the mathematical model more realistic one should take into account the fact that the dried material has rather viscoelastic properties than elastic ones, especially at the beginning of drying process [8,9,11,12]. In authors opinion, the viscoelasticity well reflect the real behavior of dried clay in the first period of drying. In this paper, Hook and Maxwell constitutive equations are used to express elastic and viscoelastic properties of the dried body. The solution of the problem was obtained with the use of both Laplace transformations and the finite difference method.

The thermodynamical background of the model used in this paper is presented by Kowalski [1,2] and Kowalski and Strumillo [3]. Phase transitions inside the dried material are ignored and the whole evaporation of the moisture is assumed to proceed on the boundary of the dried material. Thus the considerations in this work are referred to the first period of drying during which the clay-like materials suffer the most shrinkeage. During this period, the material is first preheated and next the external heat supply is totally destined into water evaporation at the boundary of the body. After the short preheating period, the temperature of the body becomes constant and equal to the wet-bulb temperature in the whole cross-section. The thermal stresses are absent in this period and the shrinkage stresses caused by a non-uniform distribution of the moisture content arising.

As a result of present considerations one states that the difference between solutions of elastic and viscoelastic problems is evidenced especially in stresses. The evolution of the shrinkage stresses of the cylinder is presented in the form of graphs.

2. Model presentation

To solve the problem of convective drying of a cylinder the following assumptions have to be satisfied:

- The cylinder is made of a porous elastic or viscoelastic material, whose pores are filled with water.
- The viscoelastic behavior of the material is described by Maxwell model.
- The drying of the cylinder proceeds symmetrically with respect to cylinder axis and the cylinder is assumed to be long enough, so that only the displacement in radial direction is considered, i.e. $u_r \neq 0$, $u_z = 0$, $u_\theta = 0$.
- The boundary surface, $r = R$ is free of external loading.
- The analysis is confined to the constant drying rate period, which is characterized by the uniform temperature in the whole body, equal to the wet-bulb temperature.

* Corresponding author.
E-mail addresses: jacek.banaszak@fct.put.poznan.pl (J. Banaszak), kowal@rose.man.pl (S.J. Kowalski).

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Nomenclature

A	bulk modules of elastic deformations (N/m^2)
c_Θ	moisture content of the moisture potential (J/m^3)
K	$3K = 2M + 3A$ (N/m^2)
M	shear modules of elastic deformations (N/m^2)
u_i	displacement (m)

Greek letters

α_Θ	moisture expansion coefficient (l)
χ	$3\chi = 2\eta + 3\xi$ (N/m^2)
ϵ_{ij}	strains (l)
η	shear modules of elastic deformations (N/m^2)
Λ_{m}	moisture transport coefficient (kg s/m^2)
μ	moisture potential (J/kg)
Θ	moisture content (kg/kg)
ρ_0	dry body mass density (kg/m^3)
σ_{ij}	stresses (N/m^2)
ξ	bulk modules of viscoelastic deformations (N/m^2)

The porous cylinder subjected to a radially symmetric moisture content fields has to satisfy the conditions of symmetry which are fulfilled only then if the shear stresses (σ_{ij} $(i \neq j)$) and tangential displacements (u_θ) equal zero (Fig. 1).

$$\sigma_{\mathrm{r}\theta} = \sigma_{\theta\mathrm{z}} = \sigma_{\mathrm{zr}} = 0 \quad \text{and} \quad u_\theta = 0 \tag{1}$$

The radial displacement u_{r} is a function of radius r and time t:

$$u_{\mathrm{r}} = u_{\mathrm{r}}(r, t) \tag{2}$$

The strains in cylindrical co-ordinates are:

$$\varepsilon_{\mathrm{rr}} = \frac{\partial u_{\mathrm{r}}}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{u_{\mathrm{r}}}{r}, \quad \varepsilon_{\mathrm{zz}} = 0 \tag{3}$$

The equilibrium equation in cylindrical co-ordinates reads:

$$\frac{\partial \sigma_{\mathrm{rr}}}{\partial r} + \frac{\sigma_{\mathrm{rr}} - \sigma_{\theta\theta}}{r} = 0 \tag{4}$$

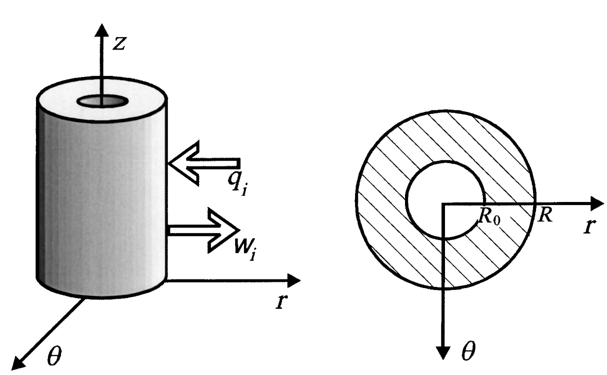

Fig. 1. Convectively dried cylinder.

The physical relations for Maxwell material type are:

$$\begin{cases} \dot{s}_{ij} + \dfrac{M}{\eta} s_{ij} = 2M\dot{e}_{ij} \\ \dot{\sigma} + \dfrac{K}{\chi}\sigma = 3K(\dot{\varepsilon} - \dot{\phi}) \end{cases} \tag{5}$$

where s_{ij} denotes the stress deviator and $\sigma\delta_{ij}$ the spherical part of the total stress tensor σ_{ij}, e_{ij} is the strain deviator and $\varepsilon\delta_{ij} = 1/3\varepsilon_{\mathrm{kk}}\delta_{ij}$—the spherical part of the total strain ϵ_{ij} and

$$\phi = \alpha_\vartheta \vartheta + \alpha_\Theta \Theta \tag{6}$$

expresses the volumetric deformation caused by the temperature and moisture content, with $\vartheta = T - T_{\mathrm{r}}$ and $\Theta = X - X_{\mathrm{r}}$, being the relative temperature and the relative moisture content, α_ϑ is the coefficient of thermal expansion and α_Θ—the coefficient of shrinkage, respectively.

The alternation of the moisture content in the dried body is described by the mass balance equation [4,10]:

$$\rho_0 \Theta = -w_{\mathrm{k,k}} \tag{7}$$

and the moisture mass transport equation, which relates the moisture flux w_{k} with the gradient of moisture potential:

$$w_{\mathrm{k}} = -\Lambda_{\mathrm{m}} \mu_{,\mathrm{k}} \tag{8}$$

where $\Lambda_{\mathrm{m}} \geq 0$ is the moisture transport coefficient. The moisture potential μ is a function of the temperature ϑ, the body volume deformation ϵ, and the moisture content Θ [4]:

$$\mu = \mu(\vartheta, \varepsilon, \Theta) = \frac{c_\vartheta \vartheta - \gamma_\Theta \varepsilon + c_\Theta \Theta}{\rho_0} \tag{9}$$

where $c_\vartheta = (\partial\mu/\partial\vartheta)_{\varepsilon,0}$ is termed the thermal coefficient of the moisture potential, $\gamma_\Theta = -\rho_0(\partial\mu/\partial\varepsilon)_{\vartheta,\Theta} = \alpha_\Theta(2M + 3A)$ can be termed as the volumetric stiffness, and $c_\Theta = c_\vartheta(\partial\mu/\partial\Theta_{\vartheta,\varepsilon}$—an averaged “Leverett function” connected with capillary rise of a wetting fluid in porous medium [5].

During the first period of drying ($\vartheta = \text{const}$) the gradient of moisture potential is:

$$\rho_0 \mu_{,\mathrm{k}} = -\gamma_\Theta \varepsilon_{,\mathrm{k}} + c_\Theta \Theta_{,\mathrm{k}} \tag{10}$$

The mass transport equation is of the form:

$$\dot{\Theta} = \frac{\Lambda_{\mathrm{m}}}{\rho_0^2} \nabla^2(-\gamma_\Theta \varepsilon + c_\Theta \Theta) \tag{11}$$

where $\nabla^2 \equiv \partial^2/\partial r + (1/r)(\partial/\partial r)$ is the Laplace operator in cylindrical co-ordinates.

The boundary conditions for the moisture transfer stimulate the symmetry of the moisture potential at the center point of the porous cylinder and convective exchange of moisture on the external surface of the moist body during drying, i.e.:

$$-\Lambda_{\mathrm{m}} \left.\frac{\partial\mu}{\partial r}\right|_{r=0} = 0 - \Lambda_{\mathrm{m}} \left.\frac{\partial\mu}{\partial r}\right|_{r=R} = \alpha_{\mathrm{m}}(\mu|_{r=R} - \mu_{\mathrm{a}}) \tag{12}$$

where α_{m} is the convective mass transfer coefficient and μ_{a} the potential of the vapor in the air (drying medium).

The initial conditions for porous cylinder being convective dried are:

$$\mu_0 = \mu(r, 0) = \text{constant} \tag{13}$$

which means that the potential of moisture is equal in whole body, before the drying process has started.

3. Displacements and stresses in viscoelastic cylinder

General solutions for displacements and stresses for the elastic cylinder dried convectively are:

$$u_\mathrm{r} = \frac{3K}{A+2M}\left(\frac{2Mr}{(2M+2A)R^2}\int_0^R \phi r\,\mathrm{d}r + \frac{1}{r}\int_0^r \phi r\,\mathrm{d}r\right) \tag{14}$$

$$\sigma_\mathrm{rr} = \frac{2M3K}{A+2M}\left(\frac{1}{R^2}\int_0^R \phi r\,\mathrm{d}r - \frac{1}{r^2}\int_0^r \phi r\,\mathrm{d}r\right) \tag{15}$$

$$\sigma_{\theta\theta} = \frac{2M3K}{A+2M}\left(\frac{1}{r^2}\int_0^r \phi r\,\mathrm{d}r + \frac{1}{R^2}\int_0^R \phi r\,\mathrm{d}r - \phi\right) \tag{16}$$

$$\sigma_\mathrm{zz} = \frac{2M3K}{A+2M}\left(2\int_0^R \phi r\,\mathrm{d}r - \phi\right) \tag{17}$$

Maxwell physical relation (5) in Laplace transforms, assuming the initial values for the stresses and strains to be zero, takes following form:

$$\bar{\sigma}_{ij}^\mathrm{V} = \frac{s}{s+M/\eta} \times\left[2M\bar{\varepsilon}_{ij} + \frac{3K(s+(M/\eta)) - 2M(s+(K/\chi))}{s+(K/\chi)}\bar{\varepsilon} - \frac{3K(s+(M/\eta))}{s+(K/\chi)}\bar{\phi}\right] \tag{18}$$

where s is the parameter of Laplace tranformation.

The elastic Hook model in Laplace transforms is written in following way:

$$\bar{\sigma}_{ij}^\mathrm{E} = 2M\bar{\varepsilon}_{ij} + 3A\bar{\varepsilon} - 3K\bar{\phi} \tag{19}$$

It is seen that the forms of these two Eqs. (18) and (19) are similar. Due to correspondence principle (see [6,7]) the viscoelastic material constants can be represented in Laplace transforms by their elastic counterparts:

$$M^\mathrm{V} = M\frac{s}{s+(M/\eta)},\quad A^\mathrm{V} = \frac{s[K(s+(M/\eta)) - 2/3M(s+(K/\chi))]}{(s+(M/\eta))(s+(K/\chi))},\quad K^\mathrm{V} = K\frac{s}{s+(K/\chi)} \tag{20}$$

The above statements justify construction of the integrals for the viscoelastic cylinder with the use of the solution for the elastic one. The integrals are similar to that (14–17),

$$\bar{u}_\mathrm{r}^\mathrm{V} = \frac{3K^\mathrm{V}}{A^\mathrm{V}+2M^\mathrm{V}} \times\left(\frac{2M^\mathrm{V}r}{(2M^\mathrm{V}+2A^\mathrm{V})R^2}\int_0^R \bar{\phi} r\,\mathrm{d}r + \frac{1}{r}\int_0^r \bar{\phi} r\,\mathrm{d}r\right) \tag{21}$$

$$\bar{\sigma}_\mathrm{rr}^\mathrm{V} = \frac{2M^\mathrm{V}3K^\mathrm{V}}{A^\mathrm{V}+2M^\mathrm{V}}\left(\frac{1}{R^2}\int_0^R \bar{\phi} r\,\mathrm{d}r - \frac{1}{r^2}\int_0^r \bar{\phi} r\,\mathrm{d}r\right) \tag{22}$$

$$\bar{\sigma}_{\theta\theta}^\mathrm{V} = \frac{2M^\mathrm{V}3K^\mathrm{V}}{A^\mathrm{V}+2M^\mathrm{V}}\left(\frac{1}{r^2}\int_0^r \bar{\phi} r\,\mathrm{d}r + \frac{1}{R^2}\int_0^R \bar{\phi} r\,\mathrm{d}r - \bar{\phi}\right) \tag{23}$$

$$\bar{\sigma}_\mathrm{zz}^\mathrm{V} = \frac{2M^\mathrm{V}3K^\mathrm{V}}{A^\mathrm{V}+2M^\mathrm{V}}\left(2\int_0^R \bar{\phi} r\,\mathrm{d}r - \bar{\phi}\right) \tag{24}$$

The coefficients standing before parenthesis in Eqs. (22)–(24) can be written with the help of (20) in the form:

$$\frac{2M^\mathrm{V}3K^\mathrm{V}}{A^\mathrm{V}+2M^\mathrm{V}} = \frac{2M3K}{A+2M}\,\frac{s}{s+((2M/3((M/\eta)-(K/\chi)) + (A+2M)M/\eta)/A+2M)} \tag{25}$$

Thus, the stresses in viscoelastic and elastic cylinders, expressed in Laplace transforms, are related to each other as follows:

$$\bar{\sigma}_{ij}^\mathrm{V} = \frac{s}{s+a}\bar{\sigma}_{ij}^\mathrm{E} \tag{26}$$

where

$$a = \frac{2M/3((M/\eta)-(K/\chi)) + (A+2M)\,(M/\eta)}{A+2M} \tag{27}$$

is a coefficient that represents the viscosity and has the meaning of the reverse of the relaxation time. It is easy to notice that the radial displacement for viscoelastic model is the same as that for elastic one, i.e.:

$$\bar{u}_\mathrm{r}^\mathrm{V} = \bar{u}_\mathrm{r}^\mathrm{E} \quad \text{or} \quad u_\mathrm{r}^\mathrm{V} = u_\mathrm{r}^\mathrm{E} \tag{28}$$

The inverse transformation for stresses gives:

$$\sigma_\mathrm{rr}^\mathrm{V}(r,t) = \sigma_\mathrm{rr}^\mathrm{E}(r,t) - a\int_0^t \mathrm{e}^{-a(t-\tau)}\sigma_\mathrm{rr}^\mathrm{E}(r,\tau)\,\mathrm{d}\tau \tag{29}$$

$$\sigma_{\theta\theta}^\mathrm{V}(r,t) = \sigma_{\theta\theta}^\mathrm{E}(r,t) - a\int_0^t \mathrm{e}^{-a(t-\tau)}\sigma_{\theta\theta}^\mathrm{E}(r,\tau)\,\mathrm{d}\tau \tag{30}$$

$$\sigma_\mathrm{zz}^\mathrm{V}(r,t) = \sigma_\mathrm{rr}^\mathrm{E}(r,t) - a\int_0^t \mathrm{e}^{-a(t-\tau)}\sigma_\mathrm{zz}^\mathrm{E}(r,\tau)\,\mathrm{d}\tau \tag{31}$$

Note that for $a \to 0$, one obtains the solution for the elastic cylinder.

4. Calculation of moisture distribution

The distribution and evolution of moisture content is described by Eq. (11). We shall use an equivalent equation expressed by the moisture potential μ:

$$K_\mathrm{m}\nabla^2\mu = \dot{\mu} \tag{32}$$

where

$$K_{\mathrm{m}} = \frac{\Lambda_{\mathrm{m}}}{\rho_0^2}\left(c_\theta - \frac{\gamma_\Theta^2}{2M + A}\right)$$

Taking into account the boundary conditions (12) and the initial condition (13) and applying the finite difference method (Crank–Nicholson scheme) one can calculate the moisture potential distribution in several instances. The moisture content is related with the moisture potential as in formula (9).

It is visible on Fig. 2 that the region close to the boundary surface ($r = 0.015$ m) is dried relatively quickly, but the center of the cylinder reaches the reference moisture content Θ_0 in 3.5 h of drying time. Such a non-uniform distribution of the moisture content induces the drying stresses. For data given above there is no difference between the curves $\Theta(r, t)$ for the elastic and viscoelastic cylinders.

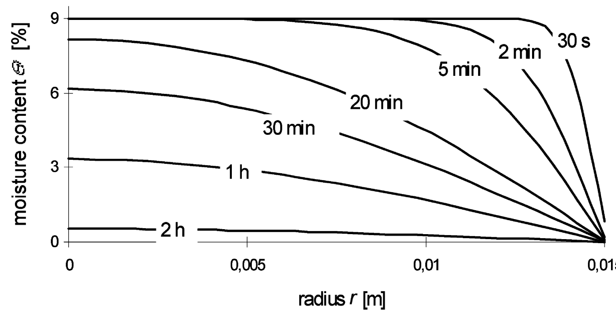

Fig. 2. Moisture content distribution in the cylinder.

5. Numerical results

The numerical problem is solved by simultaneous calculations of expressions for the radial displacement and the

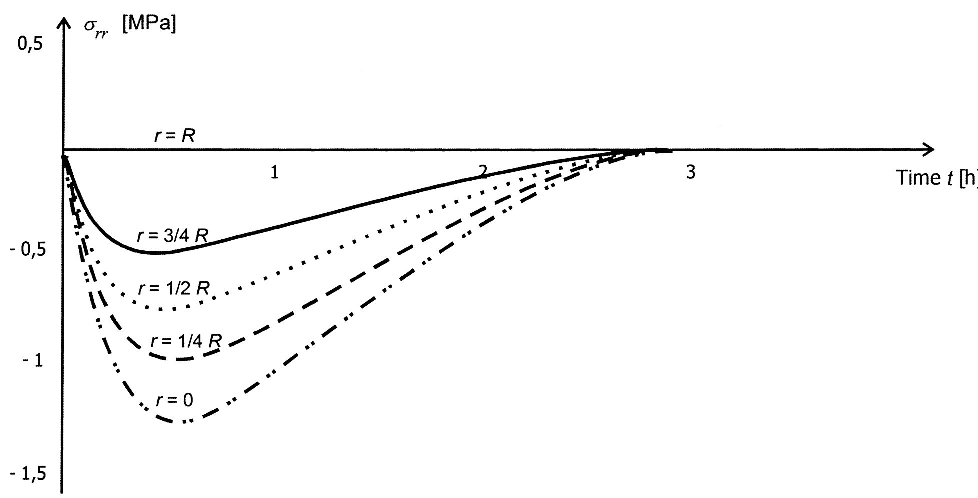

Fig. 3. Evolution of elastic radial stresses during drying process.

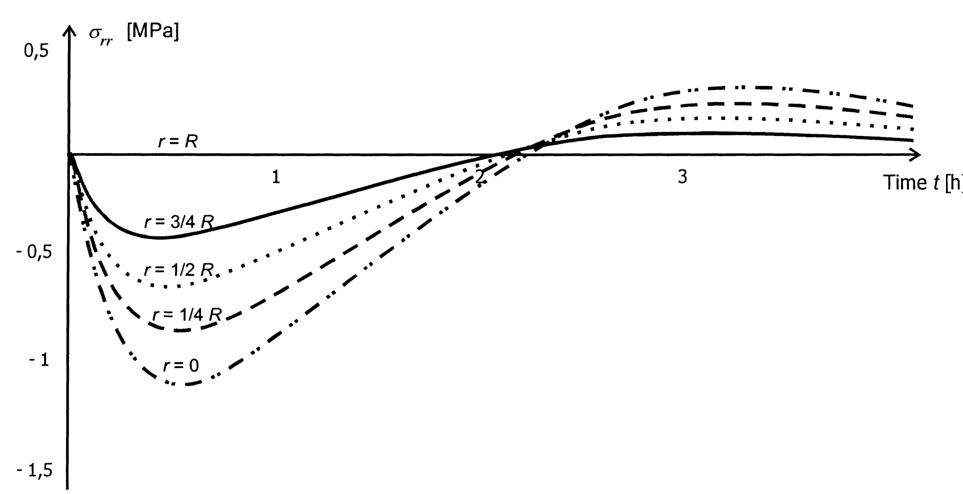

Fig. 4. Evolution of viscoelastic radial stresses during drying process.

diffusion type equation for moisture potential (32). Knowing the displacements one obtains the value of volume displacement ϵ, which is essential to find the moisture potential (10). Then, it is possible to compute elastic stresses (15–17). Finally, one can calculate the viscoelastic stresses (29–31).

The computer calculations are carried out for clay (a ceramic-like material) characterized by the following data:

$A = 10^2$ Mpa	$M = 4.25$ Mpa	$\Lambda_m = 3 \times 10^{-5}$ kg s/m^3
$\alpha_\Theta = 2.4 \times 10^{-4}$ 1	$a = 10^{-3}$–10^{-6} 1/s	$\alpha_\vartheta = 3 \times 10^{-8}\,{}^{\circ -1}$
$c_\vartheta = 2.4$ J/m^3 °	$\rho_0 = 1.2 \times 10^3$ kg/m^3	$c_\Theta = 6.6 \times 10^3$ kJ/m^3

Fig. 3 presents the evolution of the elastic radial stresses in the drying cylinder. The stresses are the highest at the body center ($r = 0$) and equal to zero on its surface ($r = R$). The stresses rapidly arise and reach their maxima after about half an hour of the drying process and then relax approaching stress free condition.

Figs. 3 and 4 allow us to compare the evolution of elastic and viscoelastic stresses in time. The viscoelastic stresses reach their maxima shortly from the beginning of the process similary to the elastic ones, but they achieve lower values. Than, the stresses starts to relax but on account of the irreversible deformations and the memory of the viscoelastic material, which remember the whole history of previous drying stresses, it comes to the change of the stress signs.

The viscosity influences significantly the magnitude of the drying induced stresses. For greater viscosity, represented by a parameter, the stress distributions are more uniform and reach lower values, Fig. 5.

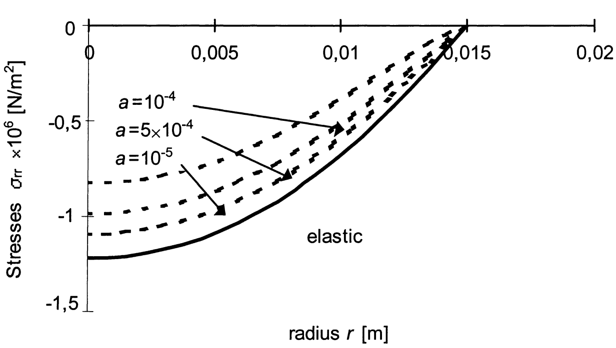

Fig. 5. Comparison of radial stress distributions in the elastic and viscoelastic cylinders.

6. Conclusions

Application of the viscoelastic constitutive model to drying processes of saturated capillary-porous materials instead of the elastic model results in the following conclusions:

- The values of stresses by viscoelastic model is lower then by the elastic model. Furthermore, the stresses reach smaller magnitude in viscoelastic materials of higher viscosity than in those of smaller viscosity.
- The viscoelastic model enables description of the phenomenon of reverse the drying induced stresses, which takes place during the drying course. The elastic model does not describe this phenomenon.

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